Word-Problem Intervention

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Word-Problem Solving Is Important

- Word-problem solving (WPS) reflects the capacity to apply mathematical ideas in everyday life and in science, technology, and engineering.

- WPS may also support advanced mathematics learning. For example, in a large national survey, U.S. Algebra I teachers considered student preparation on and the importance of 15 math skills. Teachers rated WPS highest in supporting success with algebra.

- WPS is a strong school-age predictor of employment and wages in adulthood.

WPS Difficulty is Widespread

- WP difficulty is widespread and can occur even when arithmetic skill is intact.
- Specific WP difficulty may arise because WPS engages more cognitive resources than arithmetic does.
- A common misperception among teachers is that improving children's calculation skill will result in better WPS.
- And in most classrooms and in most interventions, explicit WP strategies are not taught.

For Many Students, Arithmetic Is Not A Sufficient Pathway to WPS

- In a large randomized controlled trial conducted in 2013 with at-risk first graders, the effect of arithmetic intervention on WPS was modest (ES = 0.22) even though the effect on arithmetic was substantial (ES = 0.87).

- Also, while intervention narrowed the post-intervention arithmetic achievement gap, the WP achievement gap widened dramatically.

- This suggests that although arithmetic is foundational to WP competence, arithmetic is not a *sufficient* pathway for at-risk learners.

WPS: A Form of Text Comprehension

- This makes sense because WPs require text processing to build problem models and generate number sentences before calculations can be applied.

- Because language comprehension (LC) is necessary for text comprehension, it makes sense that LC plays a stronger role in WPS than in arithmetic.

- This has been documented when relations between LC with calculations versus with WPS are assessed in the same study.

- For this and related reasons, WPS is sometimes conceptualized and studied as a form of text comprehension.

In This Talk

- I explain a randomized controlled trial testing the effects of WP intervention that treats WPS as a form of text processing, by incorporating an explicit focus on LC.

- In explaining this RCT, I also describe the core features of our approach to WPS intervention, which relies on schema instruction.

- I end the talk by providing additional information on our WP interventions and the grade levels and types of WPs our WP interventions address.

Embedding Language Comprehension Instruction within Word-Problem Intervention

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Why Focus on WP Intervention at First Grade?

- First-grade number knowledge intervention with speeded arithmetic practice dramatically narrows the arithmetic achievement gap.

- At the same time, the WP achievement gap widens dramatically. This is problematic because
  - WPs represent a major emphasis in almost every strand of the math curriculum.
  - WP performance is a strong predictor of wages in adulthood.

- Yet, early math research has focused dominantly on number knowledge and arithmetic, with only minor focus on WPs, in the service of supporting number knowledge.

Why Focus on Language Comprehension to Improve WP Performance?

- WP solving is viewed as an interaction between language comprehension (LC) processes and math problem-solving strategies.

- When faulty problem solving is computationally modeled with math problem-solving errors versus LC errors
  - Correct problem solving depends more on LC than on math problem-solving
  - Changing wording in minor ways dramatically affects accuracy.

- Common assumption: Students have the LC for understanding problem statements and building an appropriate problem model. But for at-risk children, this assumption is shaky.

- This suggests that an instructional focus on LC processes as well as the mathematical aspects of WPS is needed.

Why *Embed* Language Instruction in WP Intervention?

(instead of providing conventional language therapy)

- Language therapy improves oral LC, but transfer to academic performance is limited, despite a strong association between LC and academic performance.

- Transfer from language therapy to academic performance may be especially difficult for at-risk children because these children often
  - Have an inadequate foundation of academic skill
  - Experience substantial challenges with transfer more generally.

- This argues for embedding language instruction in the context of direct skills intervention.

In This Study (and in our WP work generally), We Rely on Schema Instruction

- Schema instruction explicitly teaches step-by-step problem-solving strategies to reduce demands on reasoning and working memory.

- This includes strategies for understanding WPs as belonging to WP types and strategies for building WP models.

- At grade 1, we address the 3 major WP types
  - Total: 2 or more parts are combined to form a total
  - Difference: 2 quantities are compared
  - Change: an event occurs to increase or decrease a starting amount

WP Intervention

We teach the mathematical structure of each problem type.

- Role playing the problem type's central mathematical event using intact number stories (no missing quantities), concrete objects, & the child's/tutor's names

- Connecting the central mathematical event to
  - A visual schematic (into which story quantities can be entered; this is faded quickly, used only as needed)
  - A hand gesture (to quickly remind students of the schematic)
  - A problem-type sentence
    - Total: P1 + P2 = T
    - Difference: B – s = D
    - Change: St +/- C = E

- Then introducing problems (with missing quantities) using role playing, the problem type's schematic, hand gesture, and the problem type sentence.

WP Intervention

We then teach step-by-step strategies for building WP models and solving problems.

- RUN through the problem.
  - Read the problem.
  - Underline the word that indicates what the problem is mostly about.
  - Name the problem type.
- Write that problem type's sentence.
- Enter relevant quantities from the WP statement into the problem type sentence while crossing out "extra" numbers.
- Solve for the missing quantity.

Total Problem

Kathy has 5 pencils and 3 erasers. Pamela has 7 pencils. How many <u>pencils</u> do the girls have in all?

T

Combine or "Total" Problem

Kathy has 5 pencils and 3 erasers. Pamela has 7 pencils. How many pencils do the girls have in all?

T

P1 + P2 = T

Combine or "Total" Problem

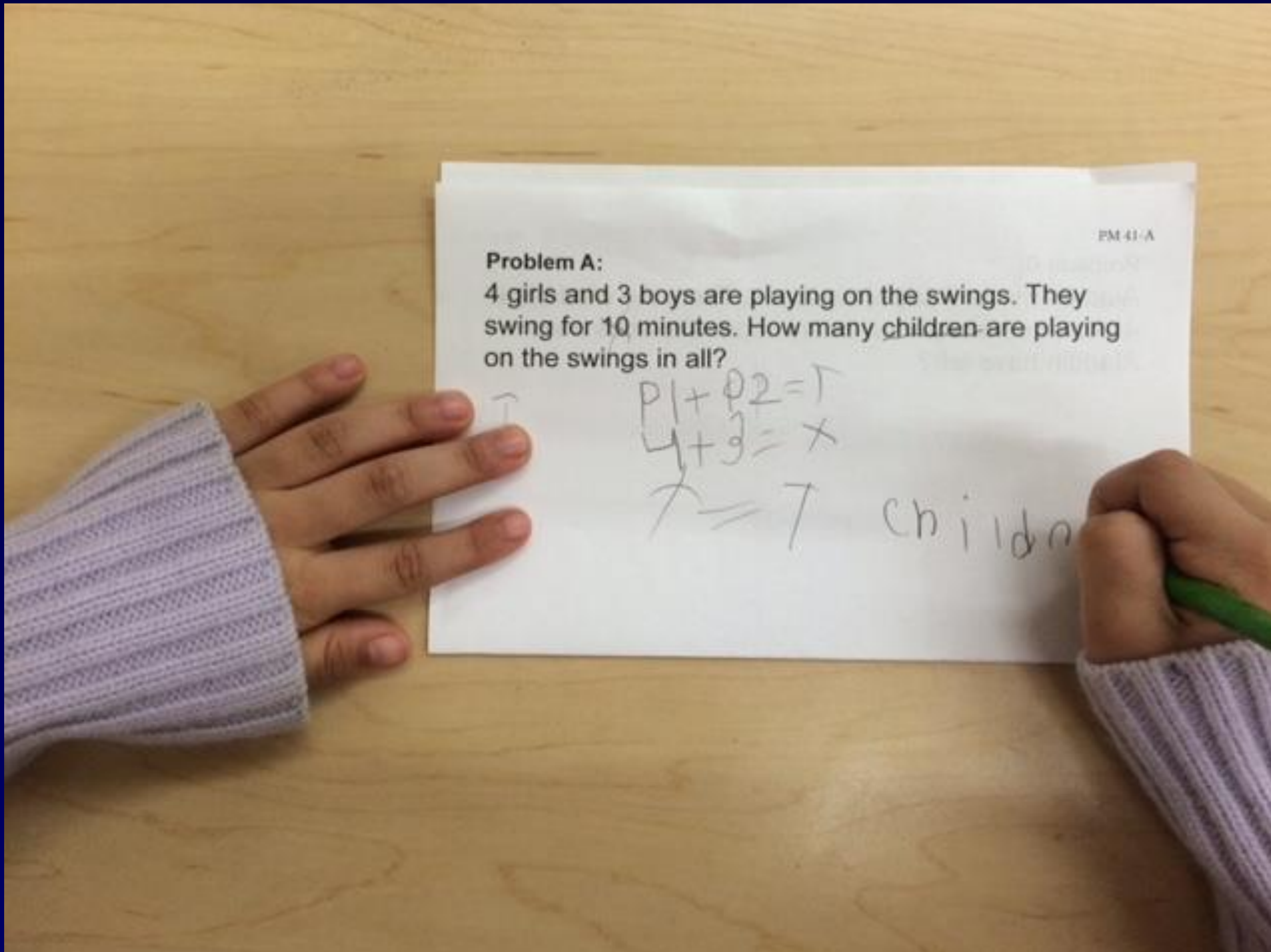
Kathy has 5 pencils and ~~3~~ erasers. Pamela has 7 pencils. How many pencils do the girls have in all?

T

P1 + P2 = T

5 + 7 = x

x = 12 pencils


PM 41-A
Problem A:
4 girls and 3 boys are playing on the swings. They swing for 10 minutes. How many children are playing on the swings in all?

Embedded Instruction

WP Vocabulary and Language Constructions

- Combine Problems
  - Joining words (e.g., *altogether, in all*)
  - Superordinate categories (e.g., *dogs* + *cats* = *animals*)
- Compare Problems
  - Compare words (e.g., *more, fewer, than*, *-er* words)
  - Adjective -*er* v. noun -*er* words (e.g., *bigger* v. *teacher*)
- Change Problems
  - Cause - effect conjunctions (e.g., *then, because, so*)
  - Implicit quantity change verbs (e.g., *cost, ate, found*)
  - Time passage phrases (e.g., *3 hours later*, *the next day*)
- Confusing cross-problem constructions (e.g., *more than* v. *then … more*)
- "Tricky" labels (e.g., questions with superordinate category words, without a label, noun that's the wrong label [as in money questions])

Embedded Language Instruction

WP Vocabulary and Language Constructions

- We do NOT teach vocabulary and language construction as key words.
- Instead, we explicitly teach why searching for key words and numbers, without reading the problem and without figuring out the problem type, often produces wrong answers.
- To help children appreciate this, we have them check the work of "other children" (worked problems we've prepared). Students find errors and explain how/why errors occurred. Worked examples
  - Rely on key words to select the wrong operation
  - Misuse irrelevant numbers
  - Fail to recognize 2-step problems.

Embedded LC Instruction

Why our embedded LC instruction isn't keyword instruction

- A Total problem: *Frieda has 8 roses. Each rose is 12 inches long. Henry has 5 daffodils. Each daffodil is 4 inches long. How many flowers do the children have altogether?* P1 + P2 = T; 8 + 5 = x; 8 + 5 = 13 flowers

- This is also a Total problem: *Frieda and Henry have 13 flowers altogether. Frieda has 5 roses. How many flowers does Henry have?* P1 + P2 = T; x + 5 = 13; 13 – 5 = 8. So *altogether* doesn't identify the operation.

- This is also a Total problem: *Frieda has 8 roses. Each rose is 12 inches long. Henry has 5 daffodils. Each daffodil is 4 inches long. How many flowers do the children have?* P1 + P2 = T; 8 + 5 = x; 8 + 5 = 13 flowers. So problem-type vocabulary (*altogether, in all*) is not always provided.

- This is also a Total problem: *Frieda has 8 roses. Each rose is 12 inches long. Henry has 5 daffodils. Each daffodil is 4 inches long. Lola has 4 tulips. How many flowers do the children have altogether?* P3 + P1 + P2 = T; 8 + 5 + 4 = 17 flowers. So >2 numbers doesn't always mean irrelevant information.

Study Overview

- Risk = low arithmetic & math concepts/applications at start of 1st grade

- 400 (391 after attrition) students randomly assigned to 4 conditions
  - WP intervention
  - WP intervention + embedded LC instruction
  - Number knowledge intervention to answer the question

    Is transfer from number knowledge and arithmetic to WPs sufficient to support WP outcomes?
  - Control (school program, most with math intervention)

- Each active intervention condition
  - Lasted 15 weeks, 3 sessions per week, 30 min per session
  - Included 5 min of speeded, strategic arithmetic practice (to control of arithmetic skill)

3-Level models
accounting for schools (21) and classrooms (186)

Arithmetic Outcome

WPS[LC] = WPS = NK > C

Effect Sizes

NK	v. C = 0.59
WPS	v. C = 0.65
WPS[LC]	v. C = 0.79

WP Outcome

WP[LC] > WP > NK = C

Effect Sizes

NK	v. C	= 0.09
WP	v. C	= 1.08
WP[LC]	v. C	= 1.75

WP[LC] v. WP = 0.47 ($p < .001$)

WP Language Outcome

WP[LC] > WP = NK = C

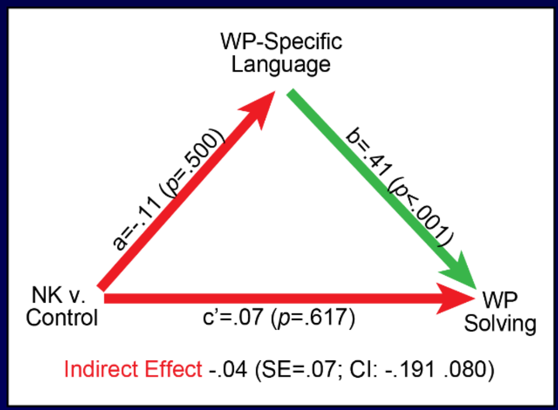
Effect Sizes

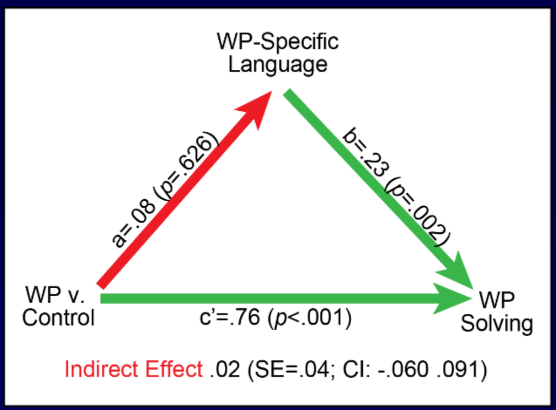
NK	v. C	= 0.17
WP	v. C	= 0.16
WP[LC]	v. C	= 0.56

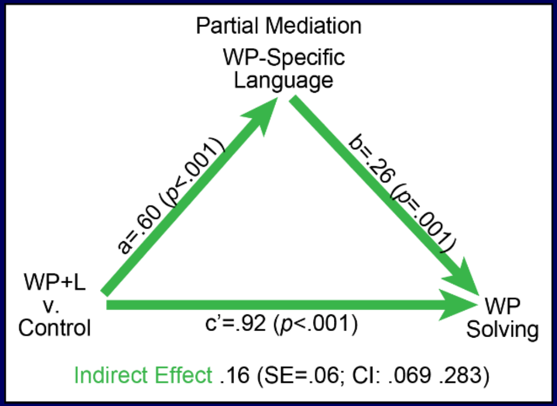
WP[LC] v. WP = 0.41

Does WP Language Mediate Condition Effects on WPS Outcomes?

Each multi-level model controls for pretest WPS and arithmetic.







Implications for Intervention

- On WPS, NK intervention does not provide added value over control (ES = 0.09), despite that NK intervention improves arithmetic skill (ES = 0.59). So transfer from arithmetic to WPS is not sufficient to support WPS development. For at-risk learners, explicit, structured WP instruction is required.

- On WPS, embedding LC instruction in WP intervention offers added value over WP intervention alone (ES = 0.41).

- This added value accrues as a function of children's improved understanding of WP language.

Next Step in This Line of Research

Does a link between WPS and RC, via LC, provide direction for understanding comorbid difficulty across WPS & RC?

2 P20 HD075443

- We are testing the effects of intervention that explicitly connects WPS and RC, via LC, in a sample of students with comorbid difficulty.

- Study conditions
  - Direct skills WP intervention with embedded WP-L instruction
  - Direct skills RC intervention with embedded RC-L instruction
  - Control

Next Step in This Line of Research

Does a link between WPS and RC, via LC, provide direction for understanding comorbid difficulty across WPS & RC?

2 P20 HD075443

- Conduct 2 tests of the "comorbidity hypothesis"
  - Whether reciprocal effects occur
    - Does WPS intervention improve RC outcomes?
    - Does RC intervention improve WP outcomes?
  - Whether LC improvement serves as a mediator these reciprocal effects, which would suggest
    - LC is a process that links WPS & RC
    - Can be effectively leveraged to improve performance in both domains in coordinated fashion.

More about
Our WP Interventions

Each WP Intervention Relies on Schema Instruction

- A *schema* is a category that encompasses similar problems. It’s a problem type.
- Schema instruction encourages students to understand problems as belonging to problem types and apply the known solution strategy for each problem type.

At the Primary Grades, Pirate Math

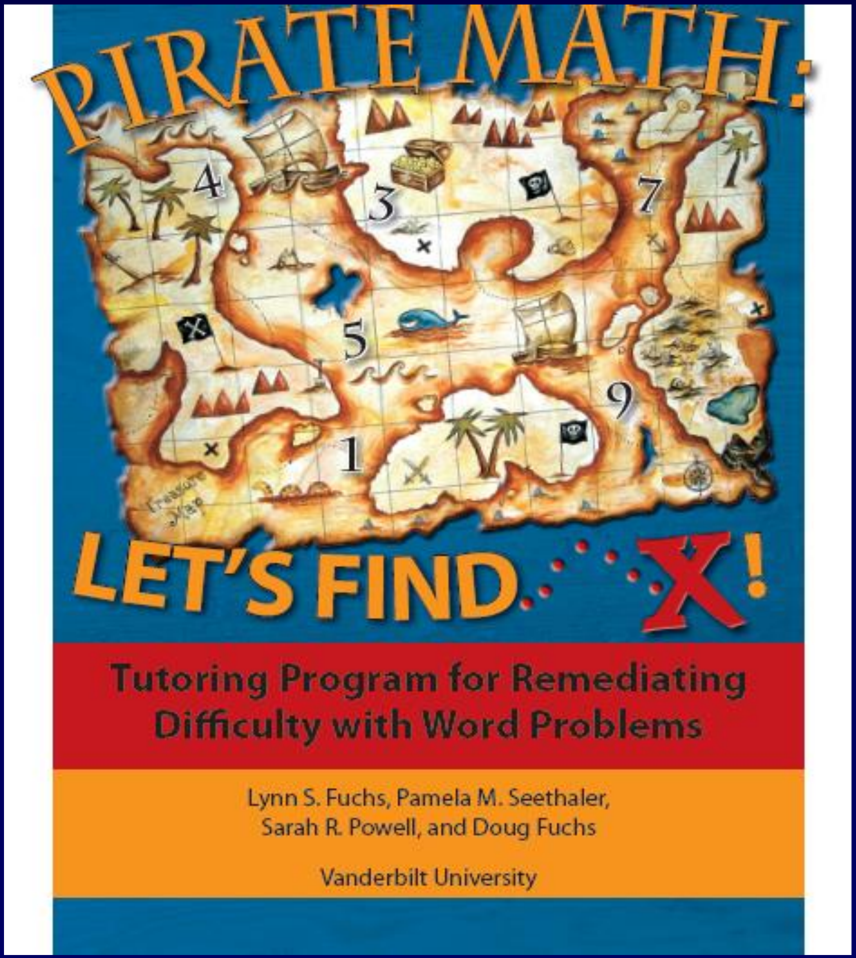
At the primary grades, we rely on variations of approach employed in the study just described.

These primary grade interventions are referred to as Pirate Math.

They address the same WP types just described.

- **Total** A scenario in which two or more parts are combined to make a whole
- **Difference** A scenario in which two amounts are compared to determine the difference
- **Change** A scenario in which a start amount changes to produce a new amount

Pirate Math



- Students taught to classify WPs into problem types; use a problem model for the identified WP type to generate a number sentence; and find the missing quantity in the number sentence -- much as pirates "find X" on a treasure map.
- Pirate theme used throughout the program for reinforcement and motivation (treasure maps, treasure chest, "X marks the spot," etc.).
- Validated in RCTs as improving students' WPS and arithmetic as well as pre-algebraic understanding.

PM Organization

- 45 lessons, organized in units

Introductory Unit (9 lessons)

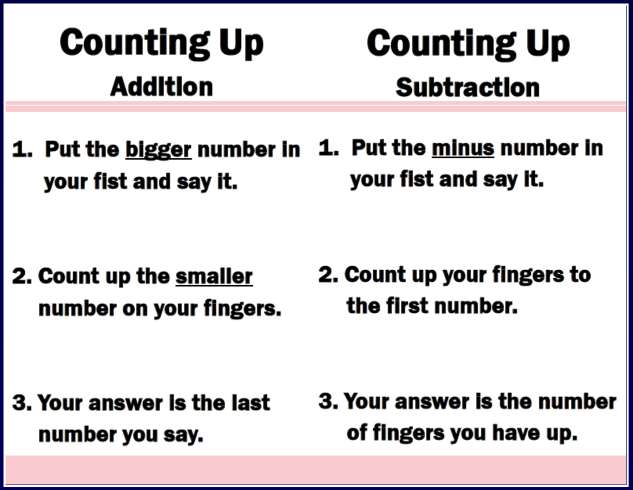
- Definition of equal sign (=), plus sign (+), minus sign (-)
- Addition/subtraction concepts (manipulatives, number line)
- Counting strategies for adding/subtracting number combinations
- Interpreting data from bar graphs and pictographs
- ___ = missing number
- How to "Find What's Missing!" when solving standard equations

8 + 9 = ___

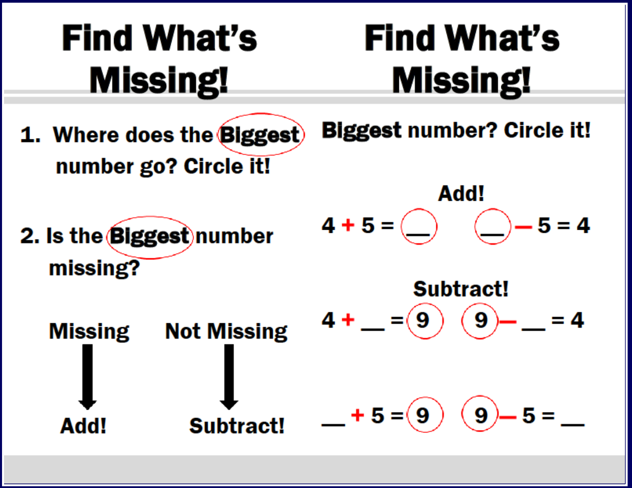
and nonstandard equations

___ + 4 = 7

10 – ___ = 1



Counting Up Addition	Counting Up Subtraction
1. Put the bigger number in your fist and say it.	1. Put the minus number in your fist and say it.
2. Count up the smaller number on your fingers.	2. Count up your fingers to the first number.
3. Your answer is the last number you say.	3. Your answer is the number of fingers you have up.



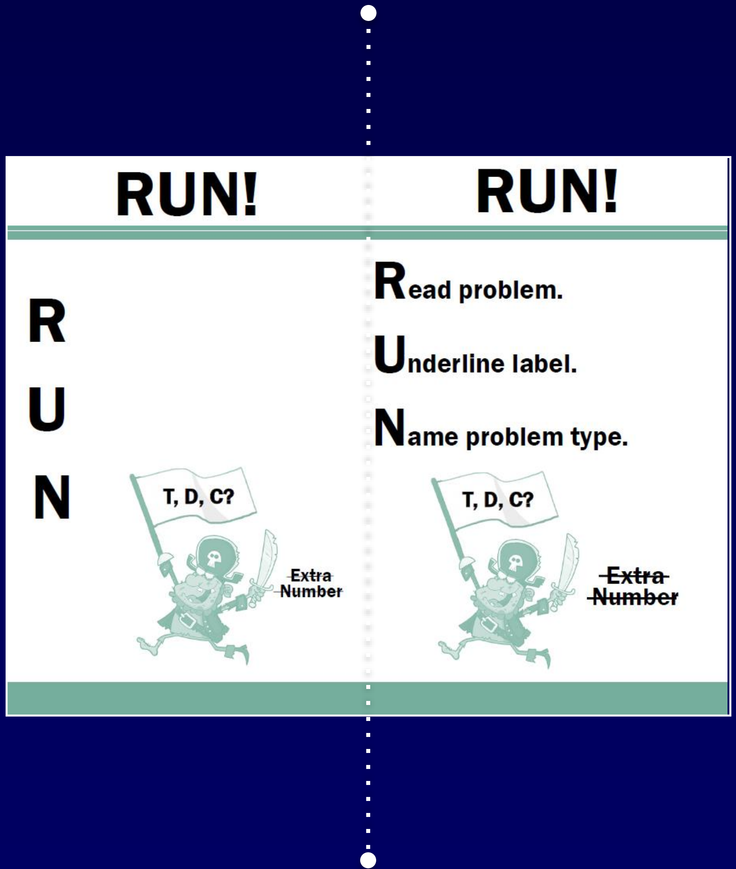
Find What's Missing!	Find What's Missing!
1. Where does the Biggest number go? Circle it!	Biggest number? Circle it!
2. Is the Biggest number missing?	Add! 4 + 5 = ___ ___ – 5 = 4
Missing → Add! Not Missing → Subtract!	Subtract! 4 + ___ = 9 9 – ___ = 4
	___ + 5 = 9 9 – 5 = ___

PM Program Organization

Once students complete introductory lessons, tutors teach students to "**RUN!**" through a word problem.

- Provides students the first set of steps for approaching the word problems.
- Serves to slow students down, think about the narrative, and build a problem model.
- Prompts them to look for and inhibit irrelevant information.

Training posters have 2 sides. The more explicit side is used initially. The other side encourages students to become less dependent on the poster. As soon as possible, tutors remove the poster, as students continue to use strategies independently.



PM Organization

- **Total Problems** Unit (Lessons 10 – 18)

  Putting two Parts together for a Total

- **Difference Problems** Unit (Lessons 19 – 27)
  - Comparing two amounts for a Difference
- **Change Problems** Unit (Lessons 28 – 36)
  - One Start amount that increases or decreases to a new, End amount
- **Review** Unit (Lessons 37 – 45)
  - Mixed practice across all WP schemas, including **transfer features**

Across units, **transfer features** increase complexity of the problems and stress children's capacity to recognize problems as belonging to a known problem type:

- Identifying and inhibiting **irrelevant information**
- Locating relevant information from **bar graphs and pictographs**
- Solving for missing information in all locations **(nonstandard equations**)
- Solving problems that require **more than one step**

3617
PMW20

T A. Webster made $42 from mowing lawns and $26 from walking dogs. How much money did he make in all?

P1 + P2 = T
42 + 26 = 68

$$\begin{array}{r} 42 \\ +26 \\ \hline 68 \end{array}$$

68$

Difference and **Change WPs**

Difference

B – s = D

(**B**igger amount) – (**s**maller amount) = **D**ifference

Pamela is 8 years old. Lynn is 15 years old. How many years younger is Pamela than Lynn?

D

B – s = D

15 – 8 = ___

7 years

Change

ST +/- C = E

(**ST**art amount) +/- (**C**hange amount) = (**E**nd amount)

Pamela had 12 M&Ms. Then, she ate some M&Ms. Now Pamela only has 4 left. How many M&Ms did Pamela eat?

C-

ST – C = E

12 – ___ = 4

8 M&Ms

Four Daily PM Lesson Activities at Grade 2

Basic Facts Fluency Building: Meet or Beat Your Score (~5 min)

- Timed addition and subtraction flash cards practice; errors are "counted up" before the next card is presented. Students graph number of correct responses.

Tutor-Led WP Lesson (~15 min)

- Tutor presents daily lesson, guiding the student to "RUN!," cross out irrelevant information, set up equation for the identified problem type (with blank as missing number), solve for the missing number, write the label, and check the reasonableness of the answer.

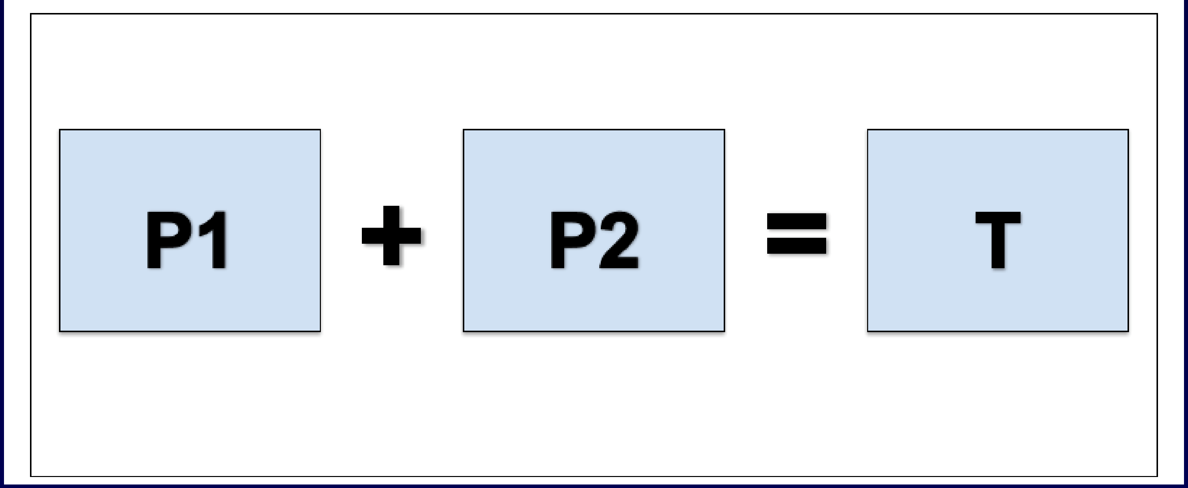
PM Game (~5 min)

- Students play games that reinforce WP instruction

Pirate Problems (~5 min)

- Timed, independent written practice with standard and nonstandard equations and with 1 word problem.

Example: **PM Game**



Students play games that reinforce WP instruction.

Total Game

Student creates a Total word problem to match a number sentence. (Tutor systematically varies the placement of the missing information.)

Example:

- Student draws 4 and 7 from pile of cards.
- Tutor places 4 on “P1” box and 7 on “T” box.
- Student makes up WP: “*Giselle and Pamela have 7 pencils in all. Giselle has 4 pencils. How many pencils does Pamela have?*”

Pirate Math Interventions

- Grades 1, 2, and 3: Interventions on whole-number total, compare, and change WPs, which also address addition and subtraction instruction

- Grade 2: Whole-class PM, with supplementary small-group PM

Our Fraction WP Interventions

- We also have a line of research on Fraction WP interventions:
  - *Fraction Faceoff!* at grade 4 (sports theme)
  - *Super Solvers* at grades 3, 4, & 5 (superhero theme)

- These fraction WP interventions build fraction understanding and procedural skill with systematic focus on fraction magnitude understanding, calculations, and WPs.

At Grades 3-5, WP Interventions with Fractions

- Grade 3: Intervention on fraction compare and change WPs, which also includes instruction on magnitude understanding and calculations

- Grades 4 and 5: Intervention on fraction compare, change, splitting, and grouping WPs, which also includes instruction on magnitude understanding and calculations

Third-Grade Fraction Intervention: 4 Lesson Activities

Problem Quest

Conceptual Instruction: Multiplication (with skip counting)

Find x by isolating the unknown

Word problems

Fraction Action

Conceptual Instruction:

Comparing Fractions

Ordering Fractions

0-1 Number Line

Super Solvers

Math Blast

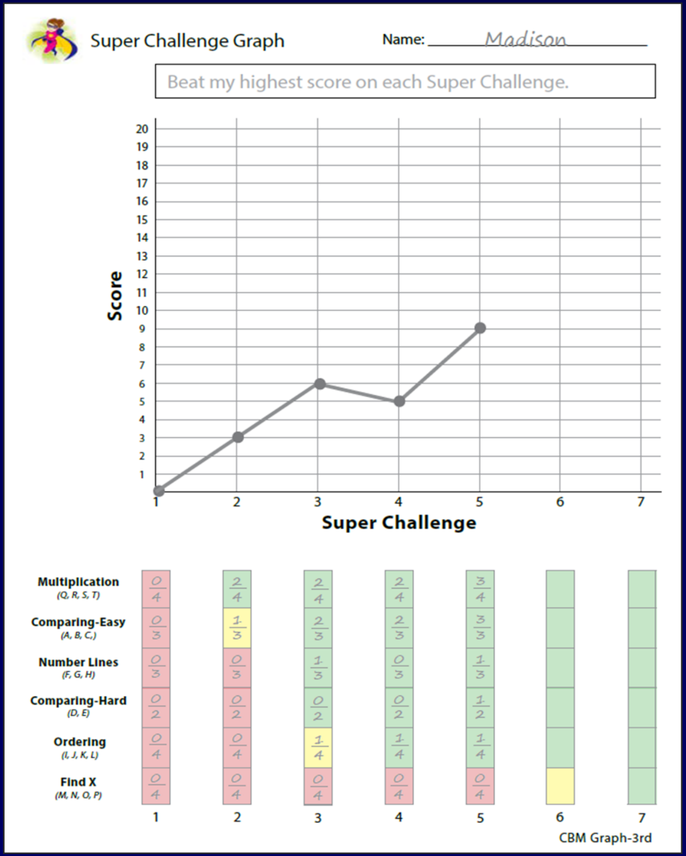
Strategic Speeded Practice:

Multiplication, Fraction equivalencies, fraction comparisons

Power Practice

Cumulative independent practice across current and previous topics

- Students also complete *The Super Challenge (*24-item CBM test) every 2 weeks. Each weekly test is an equivalent form tapping the fractions curriculum.
- Students track progress with *The Super Challenge*.



Executive Function Component

- For each upcoming *Super Challenge*, students select their overall performance goal and skills for improvement.
- Students are taught to link the importance of working hard to improving on the *Super Challenge* and classroom performance.
- Students use help strategy cards only when needed.
- Students assist their partner and ask their partner for help when needed.
- Students are encouraged to work hard and persevere with challenges, to self-regulate talking, and to share participation with their partner in the learning process.

Questions

Are standardized WP tasks available for use by school teachers to screen their students for problem-solving difficulties?

Are Tier 2 or 3 packaged intervention programs available to use in schools?

Are online resources (similar to FSU for reading research) teachers can access to teach problem solving to their students?

How early can we assess problem solving in children? Do you recommend we do it in grade 1?

Has RTI been tested in mathematics? What Tier 2&3 programs are evidence based?

For Information on Our Interventions

https://frg.vkcsites.org/

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For Information on Math Interventions Generally

- https://charts.intensiveintervention.org/aintervention

- https://www.evidenceforessa.org/